Solutio Generalis et Accurata Problematum Quamplurimorum de Motu Corporum Elasticorum incomprimibilium in Deformationibus valde Magnis

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Tametsi multis in lucubrationibus recentioribus satis pertractum est de aequilibrio corporum elasticorum vehementer deformatorum, motus talium corporum vix hactenus mathematicis attactus est. Duorum solorum problematum solutiones adhuc repertae sunt: GREEN & SHIELD [1950] gyrationem permanentem cylindri circularis artificio virium centripetarum ad quoddam problema aequilibrii reduxerunt, et KNOWLES [1960, 1], [1962] aequationem differentialem, cui oscillationes liberae talis cylindri subiciuntur, calculavit et perscrutatus est. Simplicitate peculiari autem, quae corpus incomprimibile distinguit, status aequilibrii non utitur; illa quoque permanet si status moti considerandus sit, et solutiones multorum problematum facillime reddit, utpote quas hic exponam.

Sit $\boldsymbol{x}=\boldsymbol{\chi}(\boldsymbol{X},t)$ aequatio, quae motum corporis dat; sit $\boldsymbol{F}(t)=\nabla_x\boldsymbol{\chi}=$ gradienti deformationis; sit $\ddot{\boldsymbol{x}}=\partial^2\boldsymbol{\chi}/\partial t^2=$ accelerationi; sit $\varrho=$ densitati massae; sit $\boldsymbol{b}=\boldsymbol{b}(\boldsymbol{x},t)=$ vi massali. Eo motus corporis elastici incomprimibilis subicitur sequenti aequationi generali:

$$-\nabla_x p+\nabla_x\cdot\mathfrak{g}(\boldsymbol{F}(t))+\varrho\,\boldsymbol{b}=\varrho\,\ddot{\boldsymbol{x}}, \tag{1}$$

quo $p=p(\boldsymbol{x},t)$ est pressio. Si corpus sit homogeneum, $\varrho=$const. Nunc pono motum $\boldsymbol{\chi}(\boldsymbol{X},t)$ talem esse, ut tempore t constanti deformatio $\boldsymbol{\chi}(\boldsymbol{X},t)$ statui aequilibrii corporis eadem vi massali sollicitati congruit. Hoc est, exstat talis pressio $p_0(\boldsymbol{x},t)$, ut sit

$$-\nabla_x p_0+\nabla_x\cdot\mathfrak{g}(\boldsymbol{F}(t))+\varrho\,\boldsymbol{b}=0. \tag{2}$$

Si aeq. (2) ab aeq. (1) subtrahitur, evenit

$$-\nabla_x(p-p_0)=\varrho\,\ddot{\boldsymbol{x}}. \tag{3}$$

Ergo

$$\ddot{\boldsymbol{x}}=-\nabla_x\zeta,\qquad \varrho\,\zeta=p-p_0+\psi(t). \tag{4}$$

Hoc genus motuum vulgatum est scriptoribus de rebus hydrodynamicis: illi sunt, qui circulationem omnis circuitus materialis semper eandem conservant. Valent semper in his motibus theoremata BERNOULLII, HELMHOLZII, &c. Satis de iis agitur in tractatu nostro *De theoriis classicis camporum* [1960, 2, §105 usque ad §138].

Denominetur motus talis, qualis omni momento temporis t praebet configurationem capacem aequilibrii corporis iisdem viribus massalibus sollicitati, "motus quasi aequilibratus". Generatim motus quasi aequilibratus non congruet legibus dynamicis et proinde motus verus corporis fieri non potest, manentibus iisdem viribus massalibus. Collata analysi, quam supra exposui, evenit sequens

Theorema 1. *Sit datum corpus elasticum homogeneum et incomprimibile (utrum isotropicum necne), data vi massali* $\boldsymbol{b}(\boldsymbol{x}, t)$ *sollicitatum. Ut datus motus quasi aequilibratus huius corporis possit motus verus eiusdem corporis, necesse est et sufficit*

$$\ddot{\boldsymbol{x}} = -\nabla_x \zeta, \tag{5}$$

ubi $\zeta(\boldsymbol{x}, t)$ *functio uniformis. Si valeat aeq.* (5), *pressio veri motus est*

$$p = p_0 + \varrho\zeta + \psi(t), \tag{6}$$

ubi $p_0(\boldsymbol{x}, t)$ *pressio, quae deformationem momento temporis t aequilibrare possit.*

Corollarium 1. *Ut motus quasi aequilibratus motum verum praebeat, necesse est et sufficit, ille motus et fluidis perfectis congruet, dum nullis viribus massalibus sollicitati sint.*

Scholion. Ex hypothesi sunt vires massales $\boldsymbol{b}(\boldsymbol{x}, t)$, quibus corpus sollicitatum est, eaedem et in motu vero et in motu quasi aequilibrato. Vires superficiales intestinae, quae "conatus"[1] denominentur, non sunt iam eaedem, cum si sit

$$\boldsymbol{T}_0 = -p_0 \mathbf{1} + \mathfrak{g}(\boldsymbol{F}) \tag{7}$$

tensor conatuum in motu quasi aequilibrato, sit

$$\boldsymbol{T} = -p\,\mathbf{1} + \mathfrak{g}(\boldsymbol{F}) \tag{8}$$

tensor conatuum in motu vero, et hinc

$$\boldsymbol{T} = \boldsymbol{T}_0 - \varrho\zeta\,\mathbf{1}. \tag{9}$$

Corollarium 2. *Motus verus qui ex motu quasi aequilibrato struitur, talis est, qualis impulsu idoneo pressionis in superficie confestim in statum aequilibrii reduci potest, si* $\boldsymbol{b}$ *non ex tempore t pendet.*

Corollarium 3. *In motu vero conatus transversarii*[2] *pares ac in motu quasi aequilibrato sunt.*

Theorema 1 solutiones innumerorum problematum peculiarium patefacit. Notio moti quasi aequilibrati subicitur aequationi constitutionis corporis, hoc est, formae functionis $\mathfrak{g}$. Datae functioni $\mathfrak{g}$ congruent generatim varii motus quasi aequilibrati, quorum soli qui aeq. (5) satisfaciunt motus veri possunt. Ut semel quaepiam deformatio scita est, quae aequilibrio corporis cuiusdam datis viribus massalibus sollicitati congruet et in cuius expressione analytica adsunt aliquae constantes quaecunque, motus quasi aequilibratus ex ea struitur, si istis constantibus functiones quaecunque substituuntur. Tunc calculo facili in-

[1] Anglice: stresses.
[2] Anglice: shear stresses.

venimus hic motus utrum circulationem omnis circuitus materialis perpetuè eandem conservet an non. Primum necesse est ut sit

$$\nabla_x \times \ddot{\boldsymbol{x}} = 0; \tag{10}$$

ergo est certe $\ddot{\boldsymbol{x}} = -\nabla_x \zeta(\boldsymbol{x}, t)$, secundo autem necesse est (etiam sufficit) ut sit ζ functio uniformis.

In his praesentibus dabo unum solum exemplum usus huius theorematis, quod est inventio omnium motuum quasi aequilibratorum, qui corpori elastico cuicunque, isotropico et homogeneo, nullis viribus massalibus $\boldsymbol{b}$ sollicitato congruunt. Corpus in tali statu motus necessarie omni momento temporis t in configuratione versatur, quae statui aequilibrii cuiuscunque corporis huismodi solis viribus superficialibus idoneis sollicitati consentit. Ex theoremate autem eximio quod ERICKSEN [1954] invenit, omnis talis deformatio[1] reperitur in sequentibus quinque generibus:

Genus 1 (Deformatio homogenea).

$$\boldsymbol{x} = \boldsymbol{A}\boldsymbol{X} + \boldsymbol{c}, \qquad \det \boldsymbol{A} = 1, \tag{11}$$

ubi $\boldsymbol{x} = (x, y, z)$ positio in corpore deformato, $\boldsymbol{X} = (X, Y, Z)$ positio in statu naturali, $x, y, \ldots, Z$ coordinatae rectangulares, $\boldsymbol{A}$ matrix constans quaecunque, $\boldsymbol{c}$ vector constans quicunque.

Genus 2 (Inflexio, extensio et allapsus[2] parallelepipedonis).

$$r^2 = A(2X + D), \quad \vartheta = B(Y + E), \quad z = \frac{Z}{AB} - BCY + F, \quad AB \neq 0. \tag{12}$$

ubi r, ϑ, z coordinatae cylindropolares in corpore deformato, X, Y, Z coordinatae rectangulares in statu naturali, A, B, C, D, E, F constantes quaecunque.

Genus 3 (Correctio, extensio et allapsus sectoris tubi cylindrati).

$$x = \frac{1}{2} A B^2 R^2 + D, \qquad y = \frac{\Theta}{AB} + E, \qquad z = \frac{Z}{B} - \frac{C\Theta}{AB}, \qquad AB \neq 0, \tag{13}$$

ubi x, y, z coordinatae rectangulares positionis in corpore deformato, R, Θ, Z coordinatae cylindropolares in statu naturali, A, B, C, D, E, F constantes quaecunque.

Genus 4 (Inflatio (aut eversio), inflexio, torsio, extensio et allapsus sectoris tubi cylindrati).

$$r = \sqrt{AR^2 + B}, \quad \vartheta = C\Theta + DZ + G, \quad z = E\Theta + FZ + H, \quad A(CF - DE) = 1, \tag{14}$$

ubi r, ϑ, z et R, Θ, Z coordinatae cylindropolares, illae in corpore deformato et istae in statu naturali, A, B, C, D, E, F, G, H constantes quaecunque.

[1] Ut admonitur ERICKSEN, in demonstratione sua istius theorematis est parvula lacuna: in casibus duobus peculiarissimis non potuit demonstrare quasdam aequationes superdeterminatas revera sibi mutuo repugnare. Ad enuntiatum accuratum Theorematis 2^{di} infra oportet enim animadvertere lacunam congruentem, quam autem brevitatis gratia praetermitto.

[2] Anglice: shear.

Genus 5 (Inflatio aut eversio sphaerae).

$$r^3 = \pm R^3 + A, \qquad \vartheta = \pm\Theta + B, \qquad \varphi = \Phi + C, \tag{15}$$

ubi r, ϑ, φ et R, Θ, Φ coordinatae sphaeropolares, illae in corpore deformato et istae in statu naturali, A, B, C constantes quaecunque.

Ex animadversione supra, omnis motus quasi aequilibratus reperitur, quae congruet cuicunque corpori elastico, isotropico et homogeneo, solis viribus superficialibus sollicitato, si in loco constantium $\boldsymbol{A}, A, B, \ldots, H$ aequationibus (8) usque ad (12) substituuntur functiones quaecunque temporis t: $\boldsymbol{A}(t), A(t), B(t), \ldots, H(t)$. Celeritas et acceleratio facillime deducuntur. E.g. ex aeq. (8) invenitur

$$\ddot{\boldsymbol{x}} = \ddot{\boldsymbol{A}}\boldsymbol{X} + \ddot{\boldsymbol{c}} = \ddot{\boldsymbol{A}}\boldsymbol{A}^{-1}(\boldsymbol{x} - \boldsymbol{c}) + \ddot{\boldsymbol{c}}, \tag{16}$$

quo punctum operationem derivationis calculi differentialis significat. In hoc casu fit aeq. (7)

$$\ddot{\boldsymbol{A}}\boldsymbol{A}^{-1} = (\ddot{\boldsymbol{A}}\boldsymbol{A}^{-1})^T = (\boldsymbol{A}^{-1})^T \ddot{\boldsymbol{A}}^T \tag{17}$$

Si matrix $\boldsymbol{A}$ huic aequationi satisfaciat, et si $\det \boldsymbol{A} = 1$, talis motus homogeneus $\boldsymbol{x} = \boldsymbol{A}\boldsymbol{X} + \boldsymbol{c}$ omni corpori naturae supra stipulatae congruet.

Calculum similem sed longiorem ad inveniendos motus veros, qui in superis generibus insunt, praetermitto. Ita demonstratur sequens

Theorema 2. *Ut motus quasi aequilibratus in corpore quocunque elastico, homogeneo, incomprimibili et isotropico, motus verus fieri possit, si solae vires superficiales idoneae eo adfigantur, necesse est et sufficit, ille motus in uno generum quinque supra definitorum reperitur, et insuper:*

Genere 1^{mo} (aeq. (11)):

$$\ddot{\boldsymbol{A}}^T = \boldsymbol{A}^T \ddot{\boldsymbol{A}} \boldsymbol{A}^{-1}, \tag{18}$$

ut

$$-\zeta = \tfrac{1}{2}(\ddot{\boldsymbol{A}}\boldsymbol{A}^{-1}\boldsymbol{x}) \cdot \boldsymbol{x} + (\ddot{\boldsymbol{c}} - \ddot{\boldsymbol{A}}\boldsymbol{A}^{-1}\boldsymbol{c}) \cdot \boldsymbol{x} + \psi(t). \tag{19}$$

Genere 2° (aeq. (12)):

$A = A(t) =$ functioni cuicunque temporis,

$B^2 = K \int_0^t \frac{dt}{A} + B_0^2$, ubi K et B_0 constantes,

$$C = \begin{cases} \dfrac{\dot{B}}{B}\left(K' + K'' \displaystyle\int_0^t \frac{dt}{B^2}\right) & \text{si} \quad K \neq 0, \\ \dfrac{1}{A}\left(K' + K'' \displaystyle\int_0^t A^2\, dt\right) & \text{si} \quad K = 0, \text{ ubi } K', K'' \text{ constantes,} \end{cases} \tag{20}$$

$D = D(t) =$ functioni cuicunque temporis,

$$E = K''' \int_0^t \frac{\dot{B}}{B^2}\, dt + E_0, \text{ ubi } K''' \text{ et } E_0 \text{ constantes,}$$

$F = F(t) =$ functioni cuicunque temporis, *ut est*

$$
\begin{aligned}
-\zeta = \frac{1}{2}\left[\frac{1}{2}\frac{\ddot{A}}{A} - \frac{1}{4}\frac{\dot{A}^2}{A^2} - K'''\frac{\dot{B}^2}{B^2}\right] r^2 + \frac{1}{2}(A\dot{D})^{\cdot}\log r + \\
+ \frac{A^2\dot{D}^2}{6r^3} - \frac{1}{2}\frac{\dot{B}^2}{B^2} r^2\vartheta^2 - 2K'''\frac{\dot{B}^2}{B^2} r^2\vartheta + \frac{1}{4}\frac{K\dot{D}}{B^2}\vartheta^2 + \\
+ \frac{1}{2}KK'''\frac{\dot{D}}{D^2}\vartheta + \frac{1}{2}\frac{\dddot{B}}{\dot{B}} z^2 + \left(\ddot{F} - F\frac{\dddot{B}}{\dot{B}}\right) z + \psi(t),
\end{aligned}
\tag{21}
$$

ubi si $K = 0$ ponendum est $A(1/A)^{\cdot\cdot}$ loco coefficientis $\dddot{B}/\dot{B}$.

Genere 3° (aeq. (13)):

$$
\begin{aligned}
&A = A(t) = \text{functioni cuicunque temporis,}\\
&B = B(t) = \text{functioni cuicunque temporis,}\\
&C = A(K + K'\textstyle\int B^2 dt),\\
&D = D(t) = \text{functioni cuiunque temporis,}\\
&E = E(t) = \text{functioni cuicunque temporis,}\\
&F = F(t) = \text{functioni cuicunque temporis,}
\end{aligned}
\tag{22}
$$

ubi K, K' constantes, ut

$$
\begin{aligned}
-\zeta = \frac{1}{2}\frac{(AB^2)^{\cdot\cdot}}{AB^2}(x-D)^2 + \frac{1}{2}AB\left(\frac{1}{AB}\right)^{\cdot\cdot}(y-E)^2 + \\
+ \frac{1}{2}B\left(\frac{1}{B}\right)^{\cdot\cdot}(z-F)^2 + \ddot{D}x + \ddot{E}y + \ddot{F}z + \psi(t).
\end{aligned}
\tag{23}
$$

Genere 4° (aeq. (14)), casu primo (et generaliori), $CF \neq 0$: *Si ponantur*

$$
\alpha \equiv 1 - K\left(K' + K''\int_0^t \frac{dt}{F^2}\right), \qquad \beta \equiv K^* - K'''\frac{G}{C},
$$

$$
\gamma \equiv \dot{B} - \frac{\dot{A}}{A}B, \qquad \text{ubi } K, K', K'', K''', K^* \text{ constantes,}
$$

conditiones sequentes reperiuntur.

$$
\begin{aligned}
&A = \frac{1}{CF\alpha}, \qquad \alpha \neq 0,\\
&B = B(t) = \text{functioni cuicunque temporis,}\\
&C = K'''\int_0^t F\alpha\, dt + C_0,\\
&D = KC,\\
&E = F\left(K' + K''\int_0^t \frac{dt}{F^2}\right),\\
&F = F(t) = \text{functioni cuicunque temporis,}\\
&G = K^*\int_0^t F\alpha\, dt + G_0,\\
&H = H(t) = \text{functioni cuicunque temporis,}
\end{aligned}
\tag{24}
$$

ut

$$-\zeta = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^{\cdot} + \frac{1}{2}\frac{\dot{A}^2}{A^2} - F^2\alpha^2\beta^2\right]r^2 + \frac{1}{2}\dot{\gamma}\log r + \\ + \frac{1}{8}\frac{\gamma^2}{r^2} - \frac{1}{2}\frac{\dot{C}^2}{C^2}r^2\vartheta^2 - \frac{\dot{C}}{C}F\alpha\beta r^2\vartheta + \frac{1}{2}\gamma\frac{\dot{C}}{C}\vartheta^2 + \\ + F\alpha\beta\gamma\vartheta + \frac{1}{2}\frac{\ddot{F}}{F}z^2 + \left(\frac{\ddot{H}}{H} - \frac{\ddot{F}}{F}\right)Hz + \psi(t). \tag{25}$$

Genere 4° (aeq. (14)), casu altero, $C=0$, et hinc $ADE \neq 0$:

$A = -\dfrac{1}{DE}$,

$B = B(t) =$ functioni cuicunque temporis

$C = 0$,

$D = K''' \exp\left(-K\int_0^t E\,dt\right)$, ubi K, K''' constantes et $K''' \neq 0$,

$$E = \begin{cases} F\left(K' + K''\int_0^t \frac{dt}{F^2}\right) & \text{si } F \neq 0, \text{ ubi } K', K'' \text{ constantes}, \\ E(t) = \text{functioni cuicunque si } F = 0, \end{cases} \tag{26}$$

$F = F(t) =$ functioni cuicunque temporis,

$G = D\left(K^* + K^{**}\int_0^t \frac{E\,dt}{D}\right)$, ubi K^*, K^{**} constantes,

$H = H(t) =$ functioni cuicunque temporis,

ut

$$-\zeta = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^{\cdot} + \frac{1}{2}\frac{\dot{A}^2}{A^2} - K^{**2}E^2\right]r^2 + \frac{1}{2}\dot{\gamma}\log r + \\ + \frac{1}{8}\frac{\gamma^2}{r^2} - \frac{1}{2}EK\gamma\vartheta^2 + EK^{**}\gamma\vartheta + \\ + \frac{1}{2}\frac{\ddot{E}}{E}z^2 + \left(\frac{\ddot{H}}{H} - \frac{\ddot{E}}{E}\right)Hz + \psi(t), \tag{27}$$

ubi $\gamma \equiv \dot{B} - \dot{A}B/A$.

Genere 5° (aeq. (15)):

$$A = A(t) = \text{functioni cuicunque temporis} \\ B = \text{const.} \\ C = \text{const.}, \tag{28}$$

ut

$$-\zeta = \frac{1}{3r}\left(-\ddot{A} + \frac{1}{6}\frac{\dot{A}^2}{r^3}\right). \tag{29}$$

Scholion 1. Integrationem generalem aequationis differentialis (18), cui satisfacere debet matrix $\boldsymbol{A}$, nondum perficere potui. At solutionem peculiarem dat omnis deformatio homogenea, quae semper pura manet, hoc est, cuius axes principales elapsu temporis non gyrant. Si enim deformatio semper pura est, coordinatas rectangulares fixas legere possumus, ut fit

$$\text{tum } \boldsymbol{A} = \begin{Vmatrix} \alpha(t) & 0 & 0 \\ 0 & \beta(t) & 0 \\ 0 & 0 & \gamma(t) \end{Vmatrix} \quad \text{tum } \ddot{\boldsymbol{A}} = \begin{Vmatrix} \ddot{\alpha}(t) & 0 & 0 \\ 0 & \ddot{\beta}(t) & 0 \\ 0 & 0 & \ddot{\gamma}(t) \end{Vmatrix}. \tag{30}$$

Hae formulae plane solutionem aeq. (15) dant, sed non omnem solutionem. Ut his solutionibus forma invarians detur, sit $\boldsymbol{A}=\boldsymbol{R}\boldsymbol{S}$, ubi $\boldsymbol{R}$ = matrici gyrationis, $\boldsymbol{S}$ = matrici symmetrae positivae. Si $\dot{\boldsymbol{R}}=0$, et si autovectores matricis $\boldsymbol{S}$ constantes sunt, $\boldsymbol{A}$ est solutio. Id est, si solae autovalores matricis $\boldsymbol{S}$ a tempore pendent, $\boldsymbol{A}$ satisfacit aeq. (30).

Scholion 2. Generibus 2 et 3 corpus nullo momento temporis t statum naturalem occupat. His generibus motuum oscillationes liberae excluduntur.

Corollarium 1. Genere 4°, casu priori, aeqq. (14) et (24), ponantur F = const., $B=H=0$, $K=0$ (sequuntur $D=0$ et $\alpha=1$ et $A=1/F$ = const.), $K'=K''=K'''=0$ (sequuntur $E=0$, $C=C_0$, $\beta=K^*$), et $C_0=1$. Ex formulis datis evenit

$$G=K^*Ft+G_0, \tag{31}$$

hoc est, celeritas gyrationis $\omega=K^*F$, et ex aeq. (25) sequitur

$$-\varphi=-\tfrac{1}{2}\omega^2r^2+\psi(t). \tag{32}$$

Ex aeqq. (9) et (4) struitur solutio problematis de motu gyratorio cylindri, quem dederunt GREEN & ADKINS [1950].

Corollarium 2. Genere 4°, casu priori, aeqq. (14) et (24), ponantur $A=1$ (sequitur $\gamma=\dot{B}$), $K'''=0$, $C_0=1$ (sequitur $C=1$), $K=0$ (sequitur $D=0$, $\alpha=1$), $K'=K''=0$ (sequitur $E=0$) $F=1$, $K^*=G_0=0$ (sequitur $G=0$), $H=0$. Aeq. (25) dat

$$-\varphi=\frac{1}{2}\dot{\gamma}\log r+\frac{1}{8}\frac{\gamma^2}{r^2}. \tag{33}$$

Ex aeqq. (9) et (14) consequitur formula principalis, in quam analysin suam problematum duorum de oscillationibus tuborum exstruxit KNOWLES [1960, 1], [1962].

Scholion 3. Genere 4°, casu priori, sint $\alpha=1$ et

$$F=F_0+\Phi(t), \qquad F_0>0, \tag{34}$$

ubi $\Phi(t)$ functio oscillatoria, $|\Phi(t)| \leqq F^* < F_0$. Ex aeq. (24) sequitur

$$\begin{aligned} C &= K'''\int_0^t [F_0+\Phi(t)]\,dt + C_0 \\ &= K'''\left[F_0t+\int_0^t \Phi(t)\,dt\right]+C_0. \end{aligned} \tag{35}$$

Cum sit autem $\left|\int_0^t \Phi(t)\,dt\right| \leqq F^*t$, sequitur $C\to\infty$ cum $t\to\infty$. At est $D=KC_0$. Functio D igitur functio limitata *non est*, si $K \neq 0$, et oscillatio torsionalis in hoc genere motuum *non datur*. Causa, cur oscillatio pure torsionalis non fieri potest, facile ex principiis mechanicis intellegitur. In tali motu celeritas gyrationis $r\dot{\vartheta}$ cresceret cum cresceret coordinata Z; cresceret igitur vis centripeta, ut corpus iam non cylindratum manere posset. Si vero cylindrus in oscillationibus torsionalibus movere incipiat, debeant tumefacere et contrahi alternatim anuli, qui non in quiete permaneant, et eo magis, quo distent a plano $Z=0$. Talis motus in genere 4° non inest[1].

Scholion generale. Theoremata hic adhibita facilia demonstratu sed non levia. Theorema 1 neque in corporibus elasticis comprimibilibus neque in corporibus incomprimibilibus at non elasticis generatim valet. Extendit autem per corpora incomprimibilia, quorum aequationes constitutionis sub hanc formam complectuntur:

$$\boldsymbol{T} = -p\mathbf{1} + \boldsymbol{\mathfrak{g}}(\boldsymbol{F}, \nabla_x \boldsymbol{F}, \nabla_x \nabla_x \boldsymbol{F}, \ldots). \tag{36}$$

Exemplum datur corpore elastico, in quo conatus momentorum[2] adsunt.

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Elencus operum citatorum

[1950] GREEN, A. E., & R. T. SHIELD: Finite elastic deformation of incompressible isotropic bodies. Proc. R. Soc. London A **202**, 407—419.

[1954] ERICKSEN, J. L.: Deformations possible in every isotropic incompressible, perfectly elastic body. Z. angew. Math. Phys. **5**, 466—489.

[1960, 1] KNOWLES, J. K.: Large amplitude oscillations of a tube of incompressible elastic material. Q. Applied Math. **18**, 71—77.

[1960, 2] TRUESDELL, C., & R. A. TOUPIN: The classical field theories. Flügge's Handbuch der Physik III/1, 226—793. Berlin-Göttingen-Heidelberg: Springer.

[1961] GREEN, A. E.: Torsional vibrations of an initially stressed circular cylinder. Problems of Continuum Mechanics (Muskhelisvili Anniv. Vol.), 148—154.

[1962] KNOWLES, J. K.: On a class of oscillations in the finite-deformation theory of elasticity. J. Applied Mech. **29**, 283—286.

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[1] Motus torsionalis qui his formulis datus est:

$$r = \mu R, \quad \vartheta = \Theta + \varepsilon\psi(R, Z, t), \quad z = \frac{Z}{\mu^2},$$

et quem ope methodi minimarum oscillationum contemplavit GREEN [1961, 1], solutionem accuratam non fert.

[2] Anglice: couple stresses.